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On the Radiative Corrections to the Pseudo-scalar Higgs Boson Mass

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Abstract

We reexamine the one-loop corrections to the mass of the pseudo-scalar Higgs boson, using the effective potential. In the absence of the chargino and neutralino contributions its mass exhibits a large scale dependence in the large $M_{1/2}$ regime, especially for values of $\tan\beta > 20$. Thus, although of electroweak origin, the heaviness of the $M_{1/2}$, in conjunction with the largeness of $\tan\beta$, makes these corrections very important for establishing a scale independent result and an unambiguous determination of the pseudo-scalar mass in this region of the parameter space.

1 Introduction

The Higgs sector of the MSSM has been put under experimental scrutiny the last few years and reliable lower bounds on the Higgs masses have been already established imposing tight constraints on supersymmetric models [1].

Radiative corrections to the Higgs masses have been extensively studied, especially for the lightest CP-even Higgs state for which a theoretical upper bound has been established of $\simeq 130$ GeV [2–4, 10, 5–9, 12, 11, 13, 14]. Among the others states the CP-odd Higgs A may play a significant role in processes in which CP is violated [15], and also in cosmology since the lightest supersymmetric particle (LSP) pair annihilation to a fermion pair through A exchange may enhance the corresponding cross section for large $tan\beta$, yielding LSP relic densities compatible with the recent astrophysical data [16]. The latter process is sensitive to the mass of the pseudo-scalar Higgs and a reliable determination of its mass is highly demanded.

Existing studies in literature discuss the radiative corrections to the pseudo-scalar Higgs mass using either the effective potential approach or by studying the loop corrections to the propagator [2–7, 12, 11, 10]. The mass in the second approach is the physical, or pole, mass $m_A(pole)$ which is certainly scale independent and differs from that calculated using the effective potential denoted by m_A. The two masses are related by [3, 5]

$$m_A^2(pole) = m_A^2 + \Pi_{AA}(0) - \Pi_{AA}(m_A(pole)) \, , \tag{1}$$

where $\Pi_{AA}(p^2)$ are the corrections to the pseudo-scalar propagator at momentum p. In some studies it is claimed that m_A is scale independent, however this assertion is not entirely correct since the difference $\Pi_{AA}(m_A(pole)) - \Pi_{AA}(0)$ includes small logarithmic parts, which depend explicitly on the scale Q. For completeness these should be added to m_A to render scale invariance.

In this note we consider all radiative corrections to the pseudo-scalar mass as derivatives of the one-loop effective potential. We particularly show the importance of the chargino and neutralino corrections in establishing a result which is scale independent and approximates accurately the pole mass. Long known third generation sfermion contributions are not by themselves adequate to yield a scale independent result for large $M_{1/2}$ values. The reason is that the Renormalization Group Equation (RGE) of the Higgs mixing parameter m_3^2, entering into the expression for m_A, includes terms

$$\frac{1}{16\pi^2}\,\mu\,(-6g_2^2 M_2 - \frac{6}{5}g_1^2 M_1)\,, \tag{2}$$

resulting to a scale dependence of m_3^2 not cancelled by the third generation sfermion corrections alone. In the large $M_{1/2}$ and large $\tan\beta$ regime such corrections are not numerically small.

In our approach for the determination of the pseudo-scalar mass we have duly taken into account all contributions, including the chargino/neutralino corrections, and have observed that the latter contribute significantly to the stabilization of the pseudo-scalar Higgs boson mass with respect the scale Q. If these contributions were neglected stabilization would be spoiled in the large $M_{1/2}$ region, where the running of the parameter $m_3^2(Q)$ due to the gauginos becomes important. The situation would be even more dramatic if in addition to having large $M_{1/2}$ we are in the large $\tan\beta$ regime where such deviations from stability are enhanced as being proportional to $\tan\beta$.

From Eq. (1) we see that the effective potential mass differs from the physical mass by $\Pi(0) - \Pi(m_A^2)$. In our scheme, and for a more reliable estimate of the pseudo-scalar mass, besides the chargino and neutralino corrections we have also included the contributions of the remaining sectors to m_A as well as the leading logarithmic corrections of $\Pi(0) - \Pi(m_A^2)$. Including these the resulting mass is certainly scale independent, to this loop order, and is found to be very close to the one-loop pole mass. Differences are less than 2%. In this way we have approximated satisfactorily the pole mass and avoided the complexities of calculating one-loop integrals, which are usually expressed by the Passarino–Veltman functions.

2 Radiative Corrections to the pseudo-scalar Higgs Mass

We assume a low energy supersymmetric theory described by the superpotential

$$\mathcal{W} = h_t H_2^T \epsilon\, Q U^c + h_b H_1^T \epsilon\, Q D^c + h_\tau H_1^T \epsilon\, L E^c + \mu H_2^T \epsilon\, H_1\,, \tag{3}$$

where the elements of the antisymmetric 2×2 matrix ϵ are given by $\epsilon_{12} = -\epsilon_{12} = -1$. In it only the dominant Yukawa terms of the third generation are assumed nonvanishing. The soft-SUSY breaking part of the Lagrangian is given by, in an obvious notation,

$$\begin{aligned}\mathcal{L}_{\text{scalar}} = \; & - \sum_i m_{\phi_i}^2 |\phi_i|^2 \; - \; (m_3^2 H_2^T \epsilon\, H_1 + \, h.c) \\ & - \; (A_t h_t H_2^T \epsilon\, Q U^c + A_b h_b H_1^T \epsilon\, Q D^c + A_\tau h_\tau H_1^T \epsilon\, L E^c + \, h.c) \\ & - \; \frac{1}{2}\,(M_1 \tilde{B}\tilde{B} + M_2 \tilde{W}^{(i)}\tilde{W}^{(i)} + M_3 \tilde{G}\tilde{G} + \, h.c.)\,.\end{aligned}$$

At the tree level the scalar potential of the theory is given by

$$\begin{aligned}V^0 \; &= \; m_1^2\,|H_1^0|^2 + m_2^2\,|H_2^0|^2 + m_3^2\,(H_1^0 H_2^0 + h.c) \\ & + \; (\frac{g^2 + g'^2}{8})(|H_1^0|^2 - |H_2^0|^2)^2\,,\end{aligned} \tag{4}$$

where $m_i^2 \equiv m_{H_i}^2 + \mu^2$. In this equation we have only kept the part that depends exclusively on the neutral components of the Higgs fields $H_{1,2}^0$ which is relevant for the electroweak symmetry breaking.

The one-loop correction to the effective potential is

$$\Delta V^1 = \frac{1}{64\pi^2} \sum_J (-1)^{2s_J} (2s_J+1)\, m_J^4 \left(\ln \frac{m_J^2}{Q^2} - \frac{3}{2} \right) , \tag{5}$$

where m_J are field dependent masses and s_J we denotes the spin of the J-particle.

The minimization of the one-loop corrected effective potential $V^1 \equiv V^0 + \Delta V^1$ yields the following relations

$$\sin 2\beta = -\frac{2m_3^2}{\bar{m}_1^2 + \bar{m}_2^2} \ , \quad v^2 = \frac{8}{g^2+g'^2} \frac{\bar{m}_1^2 - \bar{m}_2^2 \tan^2\beta}{\tan^2\beta - 1} , \tag{6}$$

where we have defined [1]

$$\bar{m}_i^2 \equiv m_i^2 + \Sigma_i \ , \quad \Sigma_i \equiv \left. \frac{\partial V^1}{\partial (\mathrm{Re} H_i^0)^2} \right|_{\langle H_i^0 \rangle} . \tag{7}$$

As discussed in the introduction the relation between the pole and the mass calculated using the effective potential is given by Eq. (1). In this m_A^2 is the non-zero eigenvalue of the matrix $\frac{1}{2} \left. \frac{\partial^2 V^0}{\partial \phi_i \partial \phi_j} \right|_{\langle H_i^0 \rangle}$ where $\phi_i \equiv \mathrm{Im} H_i^0$. At the tree level

$$\frac{1}{2} \left. \frac{\partial^2 V^0}{\partial \phi_i \partial \phi_j} \right|_{\langle H_i^0 \rangle} = (m_1^2 + m_2^2)\ \cos\beta \sin\beta \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} . \tag{8}$$

The matrix above has one zero eigenvalue corresponding to the mass of the Goldstone mode which is eaten up by the Z-boson. The other eigenvalue is the mass squared of the pseudo-scalar Higgs, given by $m_A^2 = m_1^2 + m_2^2$ in the lowest order.

At the one-loop the Eq. (8) is modified to

$$\begin{aligned} \frac{1}{2} \left. \frac{\partial^2 V^1}{\partial \phi_i \partial \phi_j} \right|_{\langle H_i^0 \rangle} &= (\bar{m}_1^2 + \bar{m}_2^2)\ \cos\beta \sin\beta \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} \\ &+ \frac{1}{64\pi^2} \sum_J (-1)^{2s_J} (2s_J+1)\, m_J^2 \left(\ln \frac{m_J^2}{Q^2} - 1 \right) \\ &\times \left. \begin{pmatrix} \frac{\partial^2 m_J^2}{\partial \phi_1^2} - 2 \frac{\partial m_J^2}{\partial (\mathrm{Re} H_1^0)^2} & \frac{\partial^2 m_J^2}{\partial \phi_1 \partial \phi_2} \\ \frac{\partial^2 m_J^2}{\partial \phi_2 \partial \phi_1} & \frac{\partial^2 m_J^2}{\partial \phi_2^2} - 2 \frac{\partial m_J^2}{\partial (\mathrm{Re} H_2^0)^2} \end{pmatrix} \right|_{\langle H_i^0 \rangle} , \end{aligned} \tag{9}$$

where we have used the one-loop minimization conditions of Eq. (6) as well as the relations of Eq. (7) and the one-loop corrections to the scalar potential given by the Eq. (5). This is the master formula we are going to use for the calculation of the one-loop corrections to the CP-odd Higgs boson in the effective potential approach.

Using the minimization condition relating $(\bar{m}_1^2 + \bar{m}_2^2)$ to the Higgs mixing parameter m_3^2 (see Eq. (6)), the above formula can always be cast into the form

$$\frac{1}{2} \left. \frac{\partial^2 V^1}{\partial \phi_i \partial \phi_j} \right|_{\langle H_i^0 \rangle} = -(m_3^2 + \Delta) \begin{pmatrix} \tan\beta & 1 \\ 1 & \cot\beta \end{pmatrix} , \tag{10}$$

[1] Note that in our notation $v_i \equiv \langle H_i^0 \rangle$, $v_1 \equiv \frac{v}{\sqrt{2}} \cos\beta$, $v_2 \equiv \frac{v}{\sqrt{2}} \sin\beta$, $M_W^2 = g^2(v_1^2+v_2^2)/2 = g^2 v^2/4$.

resulting to a pseudo-scalar mass squared m_A^2 given by

$$m_A^2 \;=\; -\frac{2(m_3^2+\Delta)}{\sin 2\beta}\;. \tag{11}$$

For the calculation of the CP-odd Higgs mass we thus need calculate the contribution of each particle species to Δ. The dominant third generation sfermion contributions to this quantity have been long known. The neutralino and chargino contributions are claimed to be small and hence not expected to yield substantial corrections to the pseudo-scalar mass [10]. However this may not be true when the soft gaugino mass $M_{1/2}$ is large as pointed out in the introduction. In this regime such corrections may be sizeable and should be duly taken into account.

It is well known that radiative corrections to m_A, calculated through the effective potential, are not stable with changing the renormalization scale (see for instance Ref. [18] and references therein). Empirically, one calculates m_A at an average stop mass scale, $Q_{\tilde{t}} \approx \sqrt{m_{\tilde{t}_1}\, m_{\tilde{t}_2}}$, in which case the radiative corrections of the third generation sfemions are small and can be safely neglected. In this case the pseudo-scalar mass squared is given by $m_A^2 = -2\, m_3^2(Q_{\tilde{t}})/\sin 2\beta(Q_{\tilde{t}})$, where m_3^2 is the Higgs mixing parameter. Although in principle this is correct, the contributions of charginos and neutralinos are not small at this scale, especially when $M_{1/2}$ is large. In some cases this may produce an error in the determination of its mass as large as 25%. Excluding the chargino/neutralino contribution is legitimate provided the relevant scale is not taken to be $Q_{\tilde{t}}$, but rather the average of the chargino and neutralino masses $Q_{\tilde{\chi}}$ defined by the following expression

$$Q_{\tilde{\chi}}^2 \;=\; \frac{1}{2}\,(\,\langle m_{\tilde{C}}^2\rangle \;+\; \langle m_{\tilde{Z}}^2\rangle\,)\;. \tag{12}$$

In the definition above $\langle m_{\tilde{C}}^2\rangle, \langle m_{\tilde{Z}}^2\rangle$ denote the averages of the chargino and neutralino masses squared. At this scale it is legitimate to neglect their contributions. This scale however may differ substantially from $Q_{\tilde{t}}$, when $M_{1/2}$ is large. Therefore one expects large variations of the m_A between these two scales, if only the third generation sfermion contribution is kept, resulting to a poor determination of the pseudo-scalar mass. In the following we shall calculate all corrections to the pseudo-scalar mass. The gauge and Higgs boson contributions to Δ although less important should be also included to yield a result that is scale independent and approximates satisfactorily well the pole mass.

An alternative way to calculate the quantity Δ is through the one-loop corrections to the pseudo-scalar propagator Π_{AA}. In fact following Ref. [12] we find that Δ can be expressed as

$$\Delta \;=\; s_\beta\, c_\beta\,(\,\Pi_{AA}(0) \;-\; s_\beta^2\,\frac{t_1}{v_1} \;-\; c_\beta^2\,\frac{t_2}{v_2}\,) \tag{13}$$

where $s_\beta \equiv \sin\beta$, $c_\beta \equiv \cos\beta$, and $t_{1,2}$ are the one-loop contributions to the v.e.v's $\langle H_{1,2}\rangle$. The latter can be calculated by either using the effective potential [17] or by employing diagrammatic techniques. In principle the two approaches yield identical results. However caution should be taken when gauge and Higgs boson contributions to $t_{1,2}$ are considered. These are gauge dependent and the one-loop effective potential, to our knowledge, has been only calculated in the Landau gauge. Lacking the form of the effective potential in other gauges, such as the popular 't Hooft gauge for instance, we shall rely on diagrammatic techniques to calculate the gauge and Higgs contributions to $t_{1,2}$ whenever needed.

In the following we shall discuss the various contributions to the quantity Δ. To establish our notation we briefly recall the results of Refs. [3, 5], and we first consider the third generation sfermion contributions to the CP-odd Higgs scalar. The one-loop corrections are given by Eq. (10) with Δ given by

$$\Delta^{\tilde{q}} = \frac{1}{32\pi^2}\,\mu \sum_{f=t,b,\tau} \frac{N_f\, h_f^2\, A_f}{m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2}[f(m_{\tilde{f}_1}^2) - f(m_{\tilde{f}_2}^2)].$$

In the equation above N_f is the color factor and the function $f(m^2)$ is defined by

$$f(m^2) \equiv 2m^2 \left[\ln\frac{m^2}{Q^2} - 1\right]. \tag{14}$$

The remaining sfermion contributions are zero because of the vanishing of the corresponding Yukawa couplings.

For the chargino contribution we need their field dependent mass matrix $\mathcal{M}_C$. Its squared $\mathcal{M}_C^2 = \mathcal{M}_C^\dagger \mathcal{M}_C$ is given by the following relation

$$\begin{pmatrix} M_2^2 + g^2|H_2^0|^2 & -g(M_2 H_1^0 + \mu H_2^{0*}) \\ -g(M_2 H_1^{0*} + \mu H_2^0) & \mu^2 + g^2|H_1^0|^2 \end{pmatrix}.$$

From this, after a straightforward calculation, it follows that the chargino contribution to the quantity Δ is given by

$$\Delta^{\tilde{C}} \equiv -\frac{g^2}{16\pi^2}\frac{M_2\mu}{m_{\tilde{C}_1}^2 - m_{\tilde{C}_2}^2}[f(m_{\tilde{C}_1}^2) - f(m_{\tilde{C}_2}^2)]. \tag{15}$$

The contribution of the neutralinos to Δ is less trivial to be found. Their field dependent mass matrix can be put in the following form

$$\mathcal{M}_N = \begin{pmatrix} M_1 & 0 & \frac{g' H_1^{0*}}{\sqrt{2}} & -\frac{g' H_2^{0*}}{\sqrt{2}} \\ 0 & M_2 & -\frac{g H_1^{0*}}{\sqrt{2}} & \frac{g H_2^{0*}}{\sqrt{2}} \\ \frac{g' H_1^{0*}}{\sqrt{2}} & -\frac{g H_1^{0*}}{\sqrt{2}} & 0 & -\mu \\ -\frac{g' H_2^{0*}}{\sqrt{2}} & \frac{g H_2^{0*}}{\sqrt{2}} & -\mu & 0 \end{pmatrix}. \tag{16}$$

whose squared is defined by

$$\mathcal{M}_N^2 = \mathcal{M}_N^\dagger \mathcal{M}_N. \tag{17}$$

Its field dependent eigenvalues $m^2_{\tilde{Z}_a}$, $a = 1, 2, 3, 4$ are real and are determined from the eigenvalue equation

$$h(\lambda) \equiv \det(\mathcal{M}_N^2 - \lambda I) = 0. \tag{18}$$

Obviously $m^2_{\tilde{Z}_a}$ are the physical neutralino masses squared when the Higgs fields are on their v.e.v's. The function $h(\lambda)$ can be written as

$$h(\lambda) = \lambda^4 + A\lambda^3 + B\lambda^2 + C\lambda + D\,, \tag{19}$$

where the coefficients A, B, C and D depend on the neutral Higgs fields H_i^0 .

In order to bring the second derivatives of the potential into the form of Eq. (10) we need the derivatives $\frac{\partial m^2_{\tilde{Z}_a}}{\partial(\mathrm{Re}H_i^0)^2}$ and $\frac{\partial m^2_{\tilde{Z}_a}}{\partial\phi_i\partial\phi_j}$ where ϕ_i is $\mathrm{Im}H_i^0$. The relevant derivatives entering the Eq. (9) are

$$\left.\frac{\partial m^2_{\tilde{Z}_a}}{\partial(\mathrm{Re}H_i^0)^2}\right|_{\langle H_i^0\rangle} = -\frac{1}{h'(m^2_{\tilde{Z}_a})}\left[(m^2_{\tilde{Z}_a})^3\dot{A}_i + (m^2_{\tilde{Z}_a})^2\dot{B}_i + (m^2_{\tilde{Z}_a})\dot{C}_i + \dot{D}_i\right], \tag{20}$$

and also

$$\left.\frac{\partial^2 m^2_{\tilde{Z}_a}}{\partial\phi_i\partial\phi_j}\right|_{\langle H_i^0\rangle} = -\frac{1}{h'(m^2_{\tilde{Z}_a})}\left[(m^2_{\tilde{Z}_a})^3 A''_{ij} + (m^2_{\tilde{Z}_a})^2 B''_{ij} + (m^2_{\tilde{Z}_a}) C''_{ij} + D''_{ij}\right]. \tag{21}$$

In these equations

$$\dot{A}_i \equiv \left.\frac{\partial A}{\partial(\mathrm{Re}H_i^0)^2}\right|_{\langle H_i^0\rangle} \quad \text{and} \quad A''_{ij} \equiv \left.\frac{\partial^2 A}{\partial\phi_i\partial\phi_j}\right|_{\langle H_i^0\rangle}, \tag{22}$$

and the same holds for B, C and D. In Eqs. (20,21) $h'(m^2_{\tilde{Z}_a})$ stands for the derivative

$$h'(m^2_{\tilde{Z}_a}) \equiv \left.\frac{dh(\lambda)}{d\lambda}\right|_{\lambda=m^2_{\tilde{Z}_a},\langle H_i^0\rangle}. \tag{23}$$

In these all Higgs fields are meant on their v.e.v's and hence all masses appearing are the physical tree level neutralino masses. In deriving Eq. (21) we have used the fact that

$$A'_i = B'_i = C'_i = D'_i = 0\,, \quad \text{when} \quad \phi_i = 0\,. \tag{24}$$

Using these one can find that the neutralino contribution to Δ is given by

$$\Delta^{\tilde{Z}} \equiv -\frac{g^2}{32\pi^2}\,\mu\sum_{a=1}^{4} F(m^2_{\tilde{Z}_a}) \tag{25}$$

where

$$F(m^2_{\tilde{Z}_a}) \equiv \frac{1}{h'(m^2_{\tilde{Z}_a})}\left[(m^2_{\tilde{Z}_a})^2\,c_1 + (m^2_{\tilde{Z}_a})\,c_2 + \mu^2 M_1 M_2\,c_3\right] f(m^2_{\tilde{Z}_a})\,. \tag{26}$$

The analytic expressions for the derivative $h'(m^2_{\tilde{Z}_a})$ appearing in the equation above is easily expressed in terms of the physical masses and is given by

$$h'(m^2_{\tilde{Z}_a}) \; = \; \prod_{b \neq a} (m^2_{\tilde{Z}_a} \; - \; m^2_{\tilde{Z}_b}) \qquad a, b \; = \; 1, \ldots, 4 \; . \tag{27}$$

The quantities $c_{1,2,3,}$ are given by

$$\begin{aligned} c_1 &= M_2 \, + \, \tan^2 \theta_W \, M_1 \\ c_2 &= -\tan^2 \theta_W \, M_1 \, (\mu^2 + M_2^2) \, - \, M_2 \, (\mu^2 + M_1^2) \\ c_3 &= M_1 \, + \, \tan^2 \theta_W \, M_2 \, . \end{aligned} \tag{28}$$

For a complete analysis the contributions of the gauge and Higgs bosons to the quantity Δ, although small, should be also included. Their contributions can be evaluated more easily using Eq. (13). In this case the needed corrections to the pseudo-scalar propagators and tadpoles can be read from Ref. [12]. The pseudo-scalar propagator at zero momentum transfer is given by

$$\begin{aligned} 16 \, \pi^2 \, \Pi_{AA}(0) \; &= \; \frac{g^2}{8} \Big\{ \, 2 \, \tilde{D}(H^+, W) \; + \; \frac{s^2_{\alpha\beta}}{c^2} \, \tilde{D}(H, W) \; + \; \frac{c^2_{\alpha\beta}}{c^2} \, \tilde{D}(h, W) \\ &- \; \frac{M_Z^2}{c^2} \, \big[\, c^2_{2\beta} \, (\, \bar{c}^2_{\alpha\beta} \, D(A, H) \; + \; \bar{s}^2_{\alpha\beta} \, D(A, h) \,) \, + \\ &\qquad s^2_{2\beta} \, (\, \bar{c}^2_{\alpha\beta} \, D(Z, H) \; + \; \bar{s}^2_{\alpha\beta} \, D(Z, h) \,) \, \big] \\ &+ \; \frac{1}{2c^2} \, \big[\, c_{2\beta} \, c_{2\alpha} \, (\, f(H) \; - \; f(h) \,) \; - \; 3 \, c^2_{2\beta} \, f(A) \; + \; (1 \; - \; 3 \, s^2_{2\beta} \,) \, f(Z) \, \big] \\ &+ \; \frac{1}{c^2} \, \big[\, (s^2 \, c^2_{2\beta} \; - \; c^2 \, (\, 1 \; + \; s^2_{2\beta} \,)) \, f(W) \; - \; c^2_{2\beta} \, f(H^+) \, \big] \\ &- \; 2 \, M_W^2 \, D(H^+, W) \; - \; 8 \, f(W) \; - \; \frac{4}{c^2} \, f(Z) \, \Big\}. \end{aligned} \tag{29}$$

In this expression

$$\begin{aligned} &s_{2\alpha} \; = \; \sin(2\alpha) \; , \; c_{2\alpha} \; = \; \cos(2\alpha) \quad , \quad s_{2\beta} \; = \; \sin(2\beta) \; , \; c_{2\beta} \; = \; \cos(2\beta) \\ &s_{\alpha\beta} \; = \; \sin(\alpha - \beta) \; , \; c_{\alpha\beta} \; = \; \cos(\alpha - \beta) \quad , \quad \bar{s}_{\alpha\beta} \; = \; \sin(\alpha + \beta) \; , \; \bar{c}_{\alpha\beta} \; = \; \cos(\alpha + \beta) \, . \end{aligned}$$

The functions appearing in Eq. (29) are given by

$$D(i, j) \; = \; \frac{f(i) \; - \; f(j)}{m_i^2 \; - \; m_j^2} \quad , \quad \tilde{D}(i, j) \; = \; \frac{m_i^2 \, f(i) \; - \; m_j^2 \, f(j)}{m_i^2 \; - \; m_j^2} \tag{30}$$

where $f(i) \equiv f(m_i^2)$. The function f was defined in Eq. (14). H, h, A, H^+ denote the heavy and the light CP-even Higgs, the pseudo-scalar Higgs and charged Higgs boson respectively. W, Z stand for the charged and neutral gauge bosons respectively.

The combination of the tadpoles needed to calculate the contributions of the gauge and the Higgs bosons to the quantity Δ, through Eq. (13), is given by Ref. [12]

$$b_A \quad \equiv \quad \frac{t_1}{v_1} \, s^2_\beta \; + \; \frac{t_2}{v_2} \, c^2_\beta$$

$$
\begin{aligned}
= & \frac{g^2}{32\,\pi^2} \left\{ -\frac{c_{2\beta}^2}{8c^2} f(A) \;-\; (\,\frac{1}{2} \;+\; \frac{c_{2\beta}^2}{4c^2}\,)\, f(H^+) \;+ \right. \\
+ & \frac{1}{8c^2}\,[\, s_\beta^2\,(\,c_\alpha^2 - 3\,s_\alpha^2 - s_{2\alpha}\,\tan\beta\,) \;+\; c_\beta^2\,(\,s_\alpha^2 - 3\,c_\alpha^2 - s_{2\alpha}\,\cot\beta\,)]\, f(h) \\
+ & \frac{1}{8c^2}\,[\, s_\beta^2\,(\,s_\alpha^2 - 3\,c_\alpha^2 + s_{2\alpha}\,\tan\beta\,) \;+\; c_\beta^2\,(\,c_\alpha^2 - 3\,s_\alpha^2 + s_{2\alpha}\,\cot\beta\,)]\, f(H) \\
+ & \left. \left(\frac{c_{2\beta}^2}{4c^2} - \frac{3}{2} \right) f(W) \;+\; \left(\frac{c_{2\beta}^2}{8c^2} - \frac{3}{4c^2} \right) f(Z) \right\}. \qquad (31)
\end{aligned}
$$

With this we exhaust the list of all nonvanishing contributions to the quantity Δ.

For a scale independent result we should also include the leading logarithmic contributions of the difference $\Pi_{AA}(0) - \Pi_{AA}(m_A)$ involving the renormalization scale Q. The $\log(Q^2)$ pieces of this expression, which relates the pole and effective potential masses, are listed below. These can be derived from the analytic expressions found in Ref. [12]. We write the $\log(Q^2)$ dependent part of this difference as

$$
\begin{aligned}
\Pi_{Log} \;&\equiv\; \Big[\, \Pi_{AA}(0) - \Pi_{AA}(m_A) \,\Big]_{\log Q^2} \\
&=\; (\,\sum_i X_i\,)\, \frac{m_A^2}{16\,\pi^2}\, \log\,(\,\frac{m_A^2}{Q^2}\,)\,. \qquad (32)
\end{aligned}
$$

The remaining part of $\Pi_{AA}(0) - \Pi_{AA}(m_A)$ does not involve logarithms of the scale Q^2. The nonvanishing contributions to the quantities X_i appearing in the expression above are given below

$$
\begin{aligned}
X_{fermions} \;&=\; 3\,c_\beta^2\,h_t^2 \;+\; 3\,s_\beta^2\,h_b^2 \;+\; s_\beta^2\,h_\tau^2\,, \\
X_{charginos} \;&=\; g^2\,,\; X_{neutralinos} \;=\; \frac{g^2}{\cos^2\,\theta_W}\,, \\
X_{gauge+Higgs} \;&=\; -\,g^2\,(\,\frac{3}{2} \;+\; \tan^2\beta\,)\,. \qquad (33)
\end{aligned}
$$

Within the logarithm in the Eq. (32) the pseudo-scalar mass appears. Any other mass can also lead to a scale independent result. However we have numerically verified that this choice is the most appropriate in the sense of better approximating the pole mass.

3 Results and Discussion

Following the previous discussion we see that for defining a scale independent pseudo-scalar mass we have to incorporate to the effective potential mass of Eq. (11) the logarithmic terms given by Eq. (32). Therefore we define $\tilde{m}_A^2$ as

$$
\tilde{m}_A^2 \;\equiv\; m_A^2 \;+\; \Pi_{Log}\,. \qquad (34)
$$

This is scale independent at the one-loop order and its difference from the pole mass is expected to be small. We have scanned the MSSM parameter space assuming universal boundary conditions at the Unification scale and have indeed verified that $\tilde{m}_A$

is very close to the pole mass. Differences are less than 2%. We have also seen that the logarithmic corrections Π_{Log} are small and therefore not very significant. Thus the bulk of the radiative corrections to $\tilde{m}_A$ is carried by the effective potential mass m_A. A sample result depicting very clearly the situation is displayed in the Figure 1. For the given inputs we plot the mass $\tilde{m}_A$ as function of the scale Q (dashed line) and compare it to the one-loop pole mass (dashed-dotted line). $\tilde{m}_A$ is indeed very close to the pole mass and almost independent of the scale Q for values of $Q \geq Q_{\tilde{\chi}}$. In the displayed figures the solid line corresponds to the value of the effective potential mass m_A where the contribution of charginos and neutralinos is omitted. For definiteness we shall denote this by $m_A(0)$. This is scale dependent and deviates from the pole mass. This deviation becomes significant for large values of $M_{1/2} \gg m_0$ and large $\tan\beta$. In some cases the deviation of $m_A(0)$ from the pole mass can be as large as 25% depending on the scale it is calculated. This shows the significance of the chargino and neutralino corrections in estimating the pseudo-scalar Higgs boson mass. Only at $Q \approx Q_{\tilde{\chi}}$ this coincides with the pole mass.

Figure 2 shows the same situation for the same values of the soft masses and trilinear couplings but for a smaller value of $\tan\beta$. The effect is less dramatic and the inclusion of the charginos and neutralinos, denoted by $\tilde{\chi}$ in the sequel, amounts to no more than $\approx 3\%$. In Figure 3 we have taken $M_{1/2}$ to be comparable to m_0 but the angle $\tan\beta$ is taken to be large ($= 30$). In this case the resulting $\tilde{\chi}$ corrections are of the order of $\approx 7\%$. In all figures the mass $m_A(0)$, unlike $\tilde{m}_A$, is scale dependent resulting to a poor determination of the pseudo-scalar mass. From the discussion above it becomes obvious that the effect is more pronounced in regions of the parameter space for which both $M_{1/2}$ and $\tan\beta$ are large. The reason is that in the large $\tan\beta$ regime m_A^2 is given by

$$m_A^2 \;\approx\; -\tan\beta\,(m_3^2 \,+\, \Delta)\,. \tag{35}$$

Therefore the gaugino soft terms of Eq. (2) that enter the RGE of the parameter m_3^2, which become important for large values of the parameter $M_{1/2}$, give a large scale dependence if not cancelled by corresponding terms in the corrections Δ. Failing to include the $\tilde{\chi}$ contributions results to incomplete cancellations of $\log(Q^2)$ terms in (35), and this effect is augmented for large values of $\tan\beta$. This is the reason the mass $m_A(0)$ shows this drastic scale dependence.

The masses $\tilde{m}_A$ and $m_A(0)$ are almost equal at scales comparable to $Q_{\tilde{\chi}}$ and thus both very close to the pole mass. This is expected since at $Q \approx Q_{\tilde{\chi}}$ the contribution of charginos and neutralinos are small and the Π_{Log} terms, as said before, do not contribute significantly. The smallness of the $\tilde{\chi}$'s contributions at $Q \approx Q_{\tilde{\chi}}$ is due to the fact that their masses are close to $Q_{\tilde{\chi}}$ resulting to small logarithmic corrections within the expression of the effective potential. Therefore neglecting the contribution of the charginos and neutralinos is legitimate provided that the relevant scale of the calculation is taken to be $Q_{\tilde{\chi}}$ rather than $Q_{\tilde{t}}$. These two scales however may lie quite apart. In the displayed figures the equality of the two masses is at the point where the dashed and the dashed curve cross. One sees that this scale is quite close to $Q_{\tilde{\chi}}$ whose value is shown in the figures.

Below $Q_{\tilde{\chi}}$ and $Q_{\tilde{t}}$, and as we approach M_Z, the mass $\tilde{m}_A$ shows an unstable behaviour with changing the scale Q which is however milder than that of $m_A(0)$. This was expected since for such values of Q, which lie lower than the $\tilde{t}$ and $\tilde{\chi}$ masses, large logarithms enter the one-loop corrections of the effective potential. At such scales Q only the inclusion of the two-loop corrections can ameliorate the situation and yield a stable result.

We conclude that for an unambiguous determination of the pseudo-scalar mass, as this is calculated from the effective potential, the contribution of the charginos and neutralinos should be taken into account. Their contributions is significant in regions of the parameter space for which $M_{1/2}$ and $\tan\beta$ are large. Neglecting their contributions can be legitimate only if the calculation is performed at a scale close to the average of the chargino and neutralino masses. Adding the small contributions of the remaining sectors, gauge and Higgs bosons, as well as, the small leading logarithmic corrections of the difference $\Pi(0) - \Pi(m_A^2)$, results to a mass which is scale independent and approximates accurately the pole mass.

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References

[1] Review of Particle Physics, D. E. Groom *et. al.*, Eur. Phys. J. C15 (2000) 1; "Searches for Higgs Bosons: Preliminary combined results using LEP data collected at energies up to 209 GeV", The LEP working group for Higgs boson searches, URL: http://lephiggs.web.cern.ch/LEPHIGGS/papers/.

[2] H.E. Haber and R. Hempfling, Phys. Rev. Lett. 66 (1991) 1815; Y. Okada, M. Yamagushi and T. Yanagida, Progr. Theor. Phys. 85 (1991) 1; Phys. Lett. B262(1991)54.

[3] J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B257(1991)83; ibid. B262 (1991) 477.

[4] R. Barbieri, M. Frigeni and F. Caravaglio, Phys. Lett. B258 (1991) 167; R. Barbieri and M. Frigeni Phys. Lett. B258 (1991) 395; A. Yamada, Phys. Lett. B263(1991)233.

[5] A. Brignole, Phys. Lett. 281 (1992) 284.

[6] M. Drees and M. M. Nojiri, Phys. Rev. D45 (1992) 2482.

[7] P. Chankowski, S. Pokorski and J. Rosiek, Phys. Lett. B274 (1992) 191; ibid. B281 (1992) 100; Nucl. Phys. B423 (1994) 437; A. Dabelstein, Z. Phys. C67 (1995) 495;

[8] R. Hempfling and A. Hoang, Phys. Lett. B331 (1994) 99; H. Haber, R. Hempfling and A. Hoang, Z. Phys. C75 (1997) 539.

[9] J. Casas, J. Espinosa, M. Quirós and A. Riotto, Nucl. Phys. B436 (1995) 3, E: ibid. B439 (1995) 466; M. Carena, J. Espinosa, M. Quirós and C. Wagner, Phys. Lett. B355 (1995) 209; M. Carena, M. Quirós and C. Wagner, Nucl. Phys. B461(1996)407;

[10] M. Drees, N.K. Falck and M. Glück, Phys. Rev. D33 (1986) 215; H. Eberl, Diploma thesis, TU Vienna, 1993.

[11] D. Pierce, Ph.D. Thesis, UMI-93-30687, Berkeley, 1992.

[12] J. Bagger, K. Matchev, D. Pierce and R.-J. Zhang, Nucl. Phys. B491 (1997) 3;

[13] R.-J. Zhang, Phys. Lett. B447 (1999) 89.

[14] S. Heinemeyer, W. Hollik and G. Weiglein, Phys. Rev. D58 (1998) 091701; Phys. Lett. B440 (1998) 296; S. Heinemeyer, W. Hollik and G. Weiglein, Eur. Phys. J. C9 (1999) 343; M. Carena, H. Haber, S. Heinemeyer, W. Hollik, C. Wagner and G. Weiglein, Nucl. Phys. B580 (2000) 29.

[15] A. Pilaftsis, Phys. Lett. B435 (1998) 88; A. Pilaftsis and C. E. Wagner, Nucl. Phys. B553 (1999) 3; M. Carena, J. Ellis, A. Pilaftsis and C. E. Wagner, Nucl. Phys. B586 (2000) 92; hep-ph/0009212; S. Y. Choi, M. Drees and J. S. Lee, Phys. Lett. B481 (2000) 57.

[16] A. B. Lahanas, D. V. Nanopoulos and V. C. Spanos, Phys. Lett. B464 (1999) 213; Phys. Rev. D62 (2000) 023515.

[17] R. Arnowitt and P. Nath, Phys. Rev. D46 (1992) 3981.

[18] H. Baer, C. Chen, M. Drees, F. Paige and X. Tata, Phys. Rev. Lett. 79 (1997) 986.

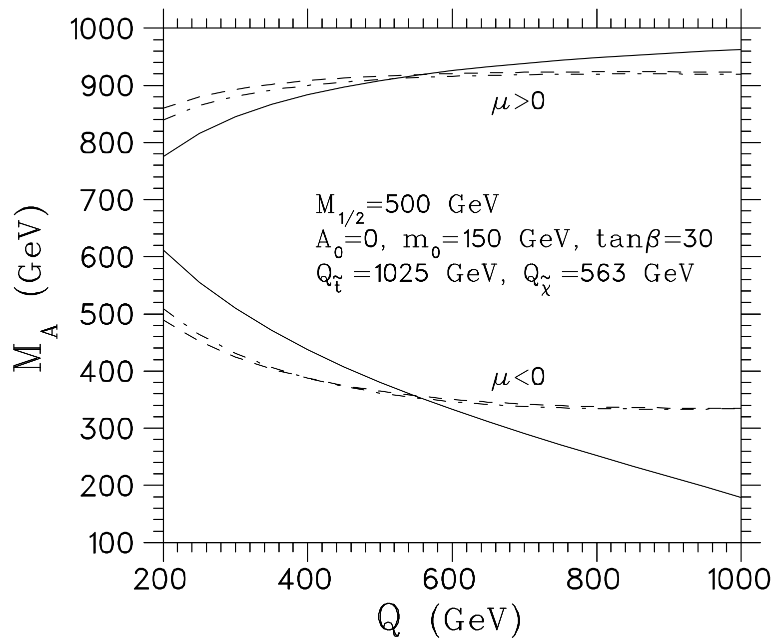

Figure 1: The pseudo-scalar mass as function of the scale Q for the inputs displayed in the figure. $Q_{\tilde{\chi}}(Q_{\tilde{t}})$ is the average chargino/neutralino (stop) mass. The solid line is the effective potential mass where only the third generation of sfermion contributes. In the dashed line the contribution of all species is taken into account as well as the the leading log wave function renormalization contributions. The dashed-dotted line is the pole mass.

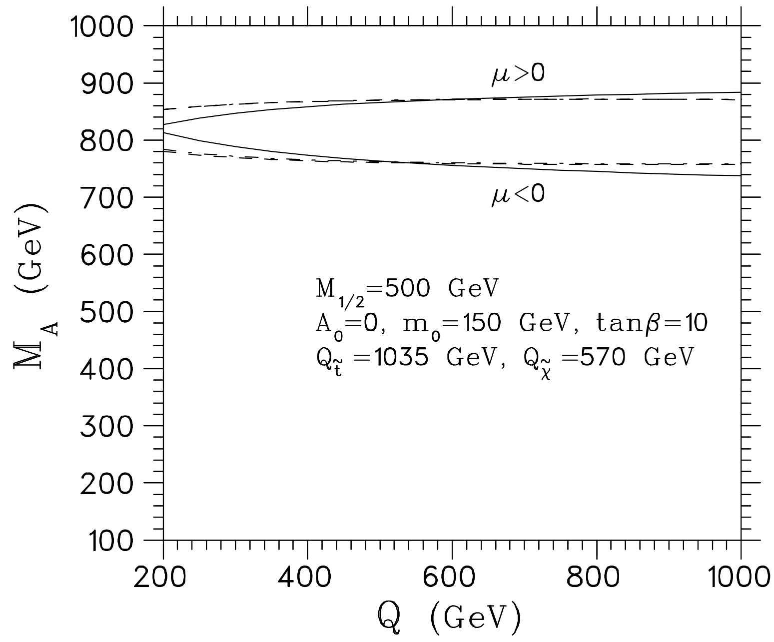

Figure 2: The same as in Figure 1 with $\tan\beta = 10$.

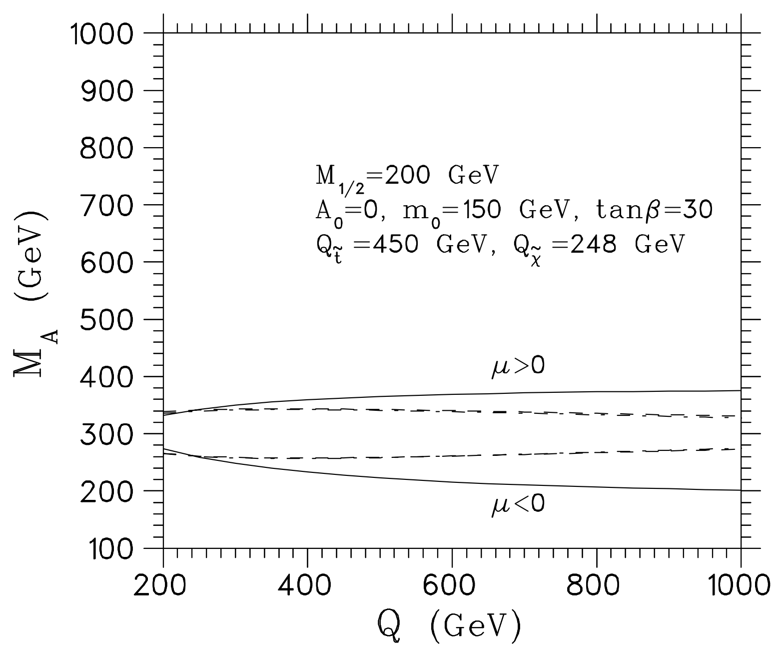

Figure 3: The same an in Figure 1 with $M_{1/2}$ to be comparable to m_0.